TrelleborgVibracoustic (Ed.)

Automotive Vibration Control Technology

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Automotive Vibration Control Technology

Fundamentals, Materials, Construction, Simulation, and Applications

Vogel Business Media

We welcome your comments and suggestions regarding the content of this reference book.
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Foreword

Modern vehicles incorporate a host of systems and components enabling safe and comfortable driving. Vibration control technology makes an especially important contribution, as it helps to isolate and dampen the unwanted noises and vibrations caused by drive systems and road irregularities. As the world's leading supplier of automotive vibration control technology, we know the challenges this poses to the developers and builders of motor vehicles. Accordingly, a team of experts at TrelleborgVibracoustic have produced a practical compendium for anyone involved in the field.

As a result of this work, we are pleased to present this reference book, *Automotive Vibration Control Technology*. Our aim has been to answer many of the questions concerning vibration control technology in vehicles – fundamental as well as topical ones. What influence do lightweight design, new drive systems and more stringent environmental demands have on vehicles' vibration behaviour? What benefits does rubber have as a material, and for which applications is polyurethane more suitable? How should a component be designed to work well within a comprehensive system? What intelligent vibration control technology solutions can meet the demand for more comfort at lower cost?

In the first part of the book we explain the fundamentals of isolating and damping vibrations in vehicles, beginning with the development of materials, moving through research, design and production processes, and ending with durability testing. The second part discusses fields of application involving powertrain and chassis technology in passenger and commercial vehicles.

We would like to thank all of the authors and their staff, as well as our development partners and customers who have all contributed to this book with their expertise and many suggestions.

We hope this book will be both stimulating and useful to our readers.

Darmstadt, July 2015

TrelleborgVibracoustic
The Management Board

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Part 1 Fundamentals

1. Vibration Control Technology for the Automotive Industry

1.1 Fundamentals and requirements of vibration control technology

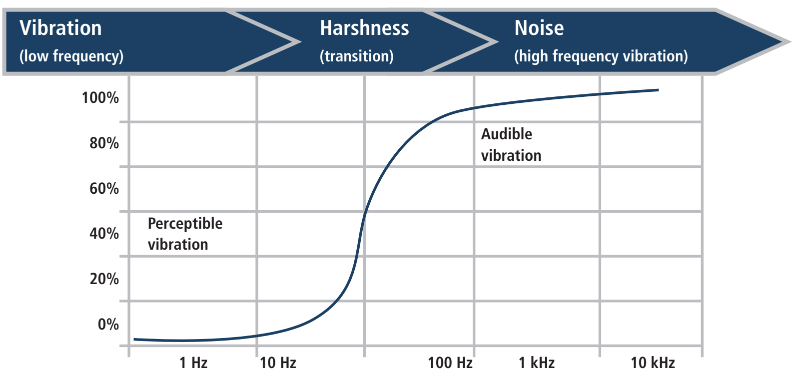
Reduced fuel consumption with improved vehicle performance, improved comfort and safety without added cost – this multifaceted challenge has motivated the automotive industry for years. Customers demand vehicles offering operating economy and value for their money, yet at the same time, vehicles that are dynamic and comfortable. Simultaneously, regulatory emissions limits become ever more stringent. The task of automotive manufacturers is to simultaneously satisfy multiple, conflicting goals. Manufacturers must produce energy-efficient, comfortable, safe and dynamic vehicles, at competitive prices. To this end, the supplier industry supports manufacturers with single-source vibration control technology solutions. Alongside engineering requirements such as lightweight design, downsizing, downspeeding, engine start-stop systems, engine cylinder deactivation and alternative propulsion technologies, rising cost pressures add another challenge to the vibration engineer's mission. Lighter vehicle structures demand special solutions, for example by integrating the masses on hand within vibration-relevant components. Downsizing of engines, downspeeding, start-stop systems and cylinder deactivation reduce weight and fuel consumption, but demand optimized engine mounting concepts, transmissions or starters – and in some cases even require additional measures such as balance shafts, dual-mass flywheels, or adaptive vibration control.

Alternative propulsion systems also demand additional measures to isolate high-frequency drivetrain noises emanating from electric motors, or annoying vibrations and noises generated by a range extender. The buyer of a premium luxury sedan does not expect to detect a difference between a four- or six-cylinder engine in terms of comfort and noise level. Beyond engineering advancements, vibration control technology is also subject to new challenges in development and production in response to market changes. In the future, volume growth will be driven more strongly by vehicles in the so-called "A" and "B" segments (US EPA minicompact and subcompact classes). And increasingly, these smaller vehicles will no longer be built in Europe. The development of innovative components for this market demands consistent application of "Design to Cost" methods, and a well grounded understanding of the needs and requirements in new markets with high growth potential, e.g. Asia. Expansion of regional development capacities will become even more important in the future.

1.2 Vibration control technology in automotive engineering

When the conversation is about ride comfort, everyone claims to know what is meant. Yet describing this "comfort" is a very complex task. Among other objectives, this volume is intended to provide the foundation to give us a better grasp of the concepts that will appear repeatedly in connection with vibration control technology. Modern passenger cars offer a high level of driving safety, combined with outstanding ride comfort. In everyday use, we are hardly aware of this – we have come to take it for granted. We would have to go back 25 years to "experience anew" a vehicle of that era, to evaluate the development progress that has been achieved since then. This progress is the end result of a steady stream of small improvements. If we could go back in time, we would once again encounter our old "friends," the idiosyncrasies of those vehicles of a bygone time. After starting, the engine reports for duty with idle shudder. Upon setting the vehicle in motion, we experience drive-off and load change bucking – annoying, impulse-like vibrations, described by some at the time as the "Bonanza effect" – for its similarity to the Cartwright clan bouncing along in their saddles in the television Western series of the same name.

Today, all of these so-called NVH (noise, vibration, and harshness) phenomena have been largely eliminated. The acronym NVH describes the totality of all occurring disturbances and their subjective perception by the vehicle occupants. These phenomena are classified according to their frequency, source, and disturbing effect, into the categories noise, vibration, and harshness (Figure 1-1). Undesirable vibrations and noises originate primarily from the combustion engine and are transmitted to the vehicle cabin as structure-borne noise and airborne noise. The suspension, too, transmits road irregularities through elastokinematic connecting elements – rubber and metal components. These are perceptible as vibrations felt at the steering wheel, seat rails, or floorpan, or in the form of undesirable noises.



Body vibration 0.5 to 5 Hz	**Jitter 15 to 40 Hz**	**Rolling noise 30 to 300 Hz**
Freeway hop 2 to 5 Hz	**Body drone 30 to 70 Hz**	**Rumble 70 to 800 Hz**
Bucking 2 to 5 Hz	**Micro-shudder 10 to 30 Hz**	**Hissing 2 to 8 kHz**
Shudder 5 to 15 Hz	**Body vibration 20 to 45 Hz**	

Figure 1-1. *Relationship between vibration frequency and subjective perception as vibration, harshness, and noise.*

For noises, the bandwidth of unpleasant effects ranges from making verbal communication difficult, to detrimental effects in listening to music, all the way to dizziness and hearing damage. The effects of more powerful and sustained vibration may range from numbness, dizziness, and loss of equilibrium to visual impairment and, in extreme cases, for example long-term exposure to construction equipment, cellular damage.

Physically perceptible, unpleasant vibrations arise in the automotive body structure and are propagated strictly as structure-borne noise – in other words, they may be felt, but are not audible. On poor road surfaces, engine shudder – a periodic vertical oscillation of the engine mass – is especially annoying. This induces a continuous shaking of the front end, which was often (erroneously) attributed to a badly tuned "shuddery" front suspension.

Subjective perception becomes problematic when vibrations can be felt as well as heard. Therefore, such disturbances should be avoided if at all possible. One example is high engine speeds, which could lead to body drone. At frequencies between 80 and 100 Hz, these are perceived as very unpleasant.

The transition regime from tactile to audible vibration or noise is designated as "harshness." This encompasses the frequency range from 15 to 100 Hz. Such disturbances are created by the road itself, and torsional vibration of the combustion engine. In the lower frequency range, perception is dominated by the tactile components; above 100 Hz, however, the audible components govern the disturbing effect.

Audible vibration at frequencies above 100 Hz are designated as "noise." Examples include rolling noise from the tires, or the high-frequency hum of an electrical machine, reminiscent of the sound made by a streetcar or tram.

In order to improve the noise comfort level, it is not sufficient to focus only on the noise range. Rather, an expanded frequency range, encompassing the entire harshness band, must be considered in order to capture all significant disturbances and eliminate them through targeted measures.

In the past few years, development engineers and vehicle acoustic experts have largely solved these vibration problems, and rubber, as an engineering material, continues to play a critical role. Vehicle acousticians have improved interior sound insulation; electronically controlled suspensions incorporating air springs permit the highest possible ride comfort without sacrificing vehicle dynamics or safety (Figure 1-2). Development engineers have optimized the engines and their mounts. Today, idle shudder or the characteristic knock of a diesel engine are things of the past. Acoustically, the modern diesel is hardly distinguishable from a gasoline engine.

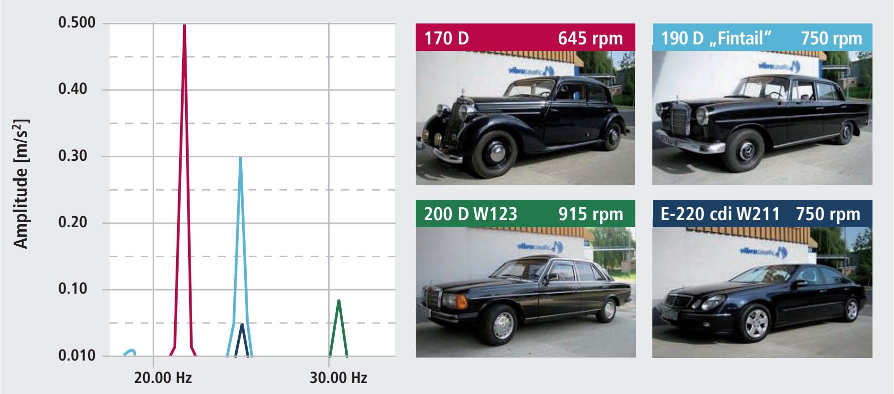


Figure 1-2. *Seat rail acceleration at idle for Mercedes-Benz vehicles of various epochs.*

2. Isolation, Damping, and Absorption

2.1 A material becomes predictable

“Rubber is black and sticky. It has an unpleasant odor and its properties change from batch to batch.” For more than 40 years, prejudices like these were used to explain both successes and failures in the rubber industry. It was difficult to make predictions, many phenomena could not be precisely explained, and there was little hope of reliably calculating the properties of a rubber mount, such as its isolation potential or its service life.

Good design solutions could only be produced by experienced “old hands” who used an empirical approach, their experience and perseverance to achieve success. Often, the physical effects of rubber in vibration isolation could only be explained on the basis of vague observations. No clear distinctions were drawn between terms such as isolation and damping. This uncertainty led to the assumption that a large rubber volume would be conducive to effective isolation. As a result, vehicle designers made considerable efforts to provide space for bulky mounts.

An engine mount from a 1985 Volkswagen Golf (Figure 2-1) is a good example: As the illustration shows, there is a large space between the inner metal sleeve, the core, and the outer steel ring. Designers expected that vibrations would be reduced on their way through the rubber; at least, this was what was hoped. Small connecting legs between the main body and the outer metal part, and a large exposed area of rubber, were intended to improve the balance between vibration energy reaching the mount and vibration transferred to the vehicle body.

Figure 2-1. *Engine mount of a Volkswagen Golf II, 1985.*

We now know that these conceptions of the physical principles of isolation were erroneous. Rubber does not isolate noise and vibration in some sort of magical way, but is a normal material that behaves in accordance with precise physical laws. Admittedly, these laws are complex. It took some time to understand the behavior of the material and to ensure that it could be described in mathematical terms. Now we are in a position to explain the term "isolation" and to predict this effect using simulation calculations with a high degree of precision at an early stage in a project.

2.2 The principles of vibration isolation

When installed, every rubber mount acts as a spring. However, a spring alone cannot provide an isolating effect. This can be explained using a simple example. A spring is positioned on a foundation and a force is applied to its loose end. The force compresses the spring which then transfers it to the foundation without any amplification or attenuation. The spring rate (i.e. whether the spring is "hard" or "soft") is immaterial. Nor does it matter whether the load on the spring is changed gradually or suddenly. The load applied to the top of the spring is transferred to the base. A spring therefore does not have an isolating effect. At most, it can only delay the transfer of a force.

Vibration can only be created by a dynamic system consisting of a spring and a mass. In the case of an engine mount, it is easy to identify the two elements. The engine is the mass and the mounts, irrespective of whether three, four or five are installed, represent the elastic springs. The result is a spring-mass system. In the case of the chassis, the situation is considerably less clear. "Masses" in this case may include the hub carrier, suspension struts and control arms, the subframe or the differential.

An oscillating system only needs excitation in order to oscillate with different displacements (amplitudes) or speeds (frequencies). Physicists were able to give a mathematically precise description of the oscillation phenomenon more than 100 years ago. The solution is presented in terms of transmission behavior (Figure 2-2) and may be explained as follows: A harmonic force of alternating direction is applied to the mass, first pulling the mass upwards and then pushing it downwards, extending or compressing the spring. The product of spring travel and stiffness is the response force transmitted to the foundation. If the force transmitted is lower than the force applied, the system has an isolating effect; if a higher force is transmitted, the system amplifies the oscillation. In other words, an oscillating system can have a damping effect without any additional damping elements; on the other hand, it can just as easily have an amplifying effect.

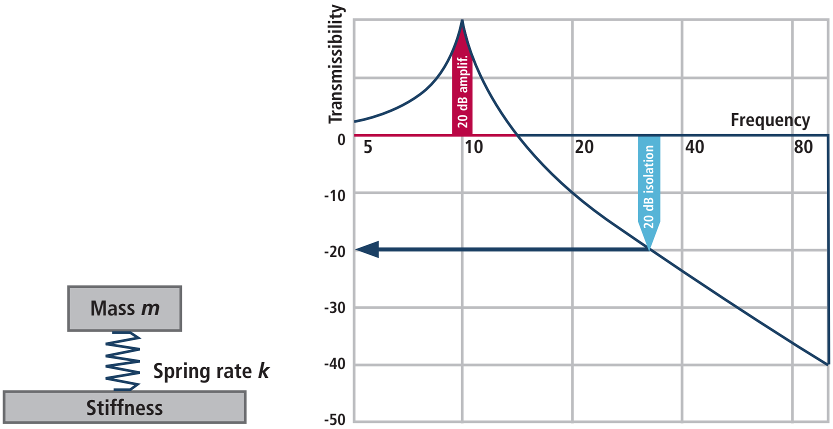


Figure 2-2. *Transmission behavior of an oscillating system.*

Which of the two possibilities occurs depends on the speed at which the force changes direction. If the direction is changed very rapidly, the mass will be unable to follow, due to inertia. It will react too slowly and will only oscillate with scarcely perceptible amplitude. Multiplied by the spring rate, these very low amplitudes result in the transmission of very low forces to the foundation (such as the body of a vehicle). If the magnitude of the exciting force remains constant but the speed at which it changes direction is reduced in very small steps, the oscillation amplitude will increase. Multiplied by the same spring rate, this means that the response force of the spring also increases. In the event that the exciting frequency matches the eigenfrequency of the oscillating system, the response force may even be many times higher than the exciting force. In this case, the spring-mass system no longer isolates the exciting force but amplifies it. With reference to an automobile, the forces that change direction at varying speed may be the combustion forces of the engine; the frequency of direction changes depends on the engine speed. Figure 2-3 gives a mathematically correct presentation of this situation. The vibration frequency is plotted on the horizontal axis, while the vertical axis above zero indicates amplification and the vertical axis below zero attenuation of the exciting force. The example shows a spring-mass system (schematically representing a simple engine mount) which has been tuned to an eigenfrequency of 10 Hz at the mass of the engine by selecting appropriate spring stiffness.

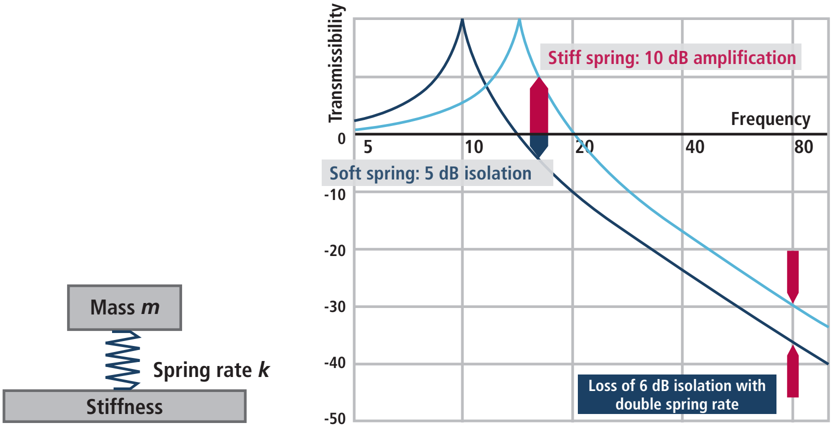


Figure 2-3. *Doubling the spring rate results in a 6 dB loss in isolation.*

Depending on the design of the engine (number and configuration of cylinders) and the operating speed, the engine excites a number of different frequencies across a wide range. At an idling speed of 600 rpm, a single-cylinder four-stroke engine generates five pulses per second, as against 20 pulses per second for a four-cylinder inline engine at the same speed.

If we assume that the engine masses are the same, the same spring (i.e. the same engine mount) would provide 2 dB amplification in the case of a single-cylinder engine and 10 dB isolation in the case of a four-cylinder engine. However, the situation is even worse: As engine speed increases, the main exciting force of the singe-cylinder engine increases, finally reaching the eigenfrequency. In our example, this effect leads to massive resonance at 1,200 rpm before the isolation range is finally reached at 1,700 rpm. In contrast, the isolating effect becomes more pronounced for the four-cylinder engine, making it more refined.

As a second example, consider two engines of different mass but identical configuration, mounted on identical springs. In the case of the first engine, its mass and elastic mounting lead to an eigenfrequency of 15 Hz. For the second engine, with half the mass of the first, this frequency slips to 21 Hz. At an idle speed of 600 rpm, the mounts of the first engine are at the limit of isolation, while for the lighter engine, the same mounting, i.e. identical mounting elements, would result in unacceptable idle shudder. The actual conditions in a car are considerably more complicated than those considered in these examples.

In its elastic mounts, an engine can move in all three directions and rotate about three axes, resulting in pitching, rolling and yawing. An engine does not have just one eigenfrequency, as assumed in the examples above for the sake of simplicity, but six different eigenfrequencies with completely different vibration modes, which may be coupled with each other. The springs (or mounts) are not installed on a rigid foundation but on elastic body structures. As a result, the vibration amplitude of the engine does not necessarily result in corresponding travel in the rubber mounts. The spring travel of the base of the mount must be taken into account with the appropriate sign, depending on the excitation frequency and the phase configuration.

The situation is further complicated by the fact that, in practical applications, a rubber mount does not react with a single “spring rate”. In simulations, the properties of a mount are not normally characterized by a single parameter but by six measured values.

In addition to the three displacement values, key rotational values plus the torsional and flexural stiffness values must be taken into account in the case of chassis bushings. The measurement of these values calls for complex instrumentation and fixtures, and represents the main challenge. It is only in a few exceptional cases that measurements produce linear force/travel characteristic curves. Typically, the measurement curves are nonlinear and measurements indicate different spring rates as a function of pre-load, test velocity, test amplitude and loading history. The spring rates of a rubber supporting spring after a few load cycles will be completely different from the values measured prior to conditioning. It may also be significant to observe that a spring element with preload in one direction will react with completely different properties in other directions. For example, a mount may become stiffer in the direction of travel if it is bearing the weight of the engine or has to support torque at different engine speeds and with different transmission ratios. Radial preload on a chassis bushing may affect the spring properties in the torsional and flexural directions.

Precise, detailed test specifications are essential for systematic component development and for comparison measurements by suppliers and customers. However, there is one complex question that a test specification cannot answer: "Will this component achieve perfect results as regards safety, comfort and durability when it is installed in the vehicle?"

This question is difficult to answer. In many applications, it is not possible to make a reliable prediction. The objective of this book is to assist in finding solutions to this problem.

2.3 Four-pole theory: an approach to describing the isolation of high frequencies

This section lays the theoretical foundations required for describing vibration transmission and isolation effects. Mechanical impedances and four-pole networks can describe the dynamic behavior of components and interfaces. On this basis, it is possible to derive a number of isolation values that are useful for the analysis and assessment of designs. The following section deals with mechanical impedances. Instead of these impedances, it is also possible to consider dynamic or apparent masses or input stiffness; the parameters can be converted into each other. *Sell* [2-1] gives a comprehensive description of the theory with examples.

2.3.1 Mechanical impedance

The mechanical impedance is the resistance of a linear elastic body to an external force. Impedance is defined as the ratio of the force applied, $\underline{F}$ to the velocity of the point to which it is applied $\underline{v}$:

$$\underline{Z} = \frac{\underline{F}}{\underline{v}}, \qquad \text{Eq. (2-1).}$$

If two bodies (e.g. masses) are moved at the same velocity at a connected point, the force applied is opposed by the sum of the impedances of the two bodies.

$$\underline{Z}_{\text{total}} = \underline{Z}_1 + \underline{Z}_2, \qquad \text{Eq. (2-2).}$$

If the same force is applied via two bodies (e.g. springs), the effective impedance is given by:

$$\frac{1}{\underline{Z}_{\text{total}}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}, \qquad \text{Eq. (2-3).}$$

Often, other parameters are considered instead of impedance. Within the frequency range, all these parameters can be converted into each other. For the determination of impedance, it is normal practice to measure force and acceleration $\underline{a}$. The impedance can then be calculated using the angular frequency ω by:

$$\underline{Z} = \frac{\underline{F}}{\underline{v}} = \frac{\underline{F}}{\underline{a}} j\omega, \qquad \text{Eq. (2-4).}$$

This formula is very useful if the input impedance of a structure is to be determined experimentally using an impulse hammer or shaker. Some formulas for calculating the impedance of ideal components are given in Table 2-1.

Table 2-1. *Impedances of idealized components.*

Component	Impedance	Symbols
Mass	$\underline{Z}_m = j\omega m$	m : mass
Spring	$\underline{Z}_k = \frac{k}{\omega m}$	k : spring constant
Viscous damper	$\underline{Z}_c = c$	c : damping coefficient

The formulas indicate that impedance is a function of frequency. To better illustrate this relationship, Figure 2-4 shows characteristic plots of impedance and dynamic mass. The expressions in parentheses indicate the relevant proportionality factors, i.e. the extent to which the value is dependent on frequency.

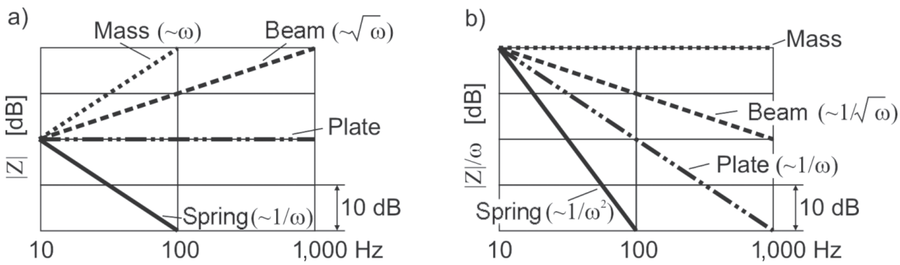


Figure 2-4. *Characteristic curves: a) impedances, b) dynamic masses.*

2.3.2 Mechanical four-pole systems

Mechanical four-pole systems were derived in the mid-20th century from the frequently used electrical four-pole system models. They represent a convenient approach to the simple presentation of relationships between mechanical components. Two inputs and two outputs are always considered. Each of the pairs consists of one force value and one displacement, velocity or acceleration value. The index 1 is used for inputs and the index 2 for outputs. Indices 12 and 21 represent transmission values. In the following paragraphs, as in the case of impedances, velocities are considered.

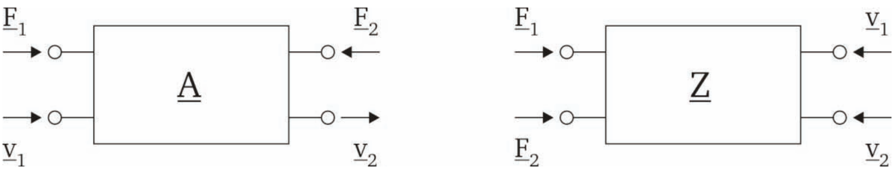


Figure 2-5. *Mechanical four-pole systems in chain form (left) and impedance form (right).*

As we will see, the chain form is especially well-suited for the calculation of series connections. Expressed as matrices, the following equations apply to the four-pole system shown in Figure 2-5:

$$\begin{bmatrix} \underline{F}_1 \\ \underline{v}_1 \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \cdot \begin{bmatrix} \underline{F}_2 \\ \underline{v}_2 \end{bmatrix} = \left[\underline{\underline{A}}\right] \cdot \begin{bmatrix} \underline{F}_2 \\ \underline{v}_2 \end{bmatrix} (\textit{chain form}), \qquad \text{Eq. (2-5).}$$

$$\begin{bmatrix} \underline{F}_1 \\ \underline{F}_2 \end{bmatrix} = \begin{bmatrix} \underline{z}_{11} & \underline{z}_{12} \\ \underline{z}_{21} & \underline{z}_{22} \end{bmatrix} \cdot \begin{bmatrix} \underline{v}_1 \\ \underline{v}_2 \end{bmatrix} = \left[\underline{\underline{Z}}\right] \cdot \begin{bmatrix} \underline{v}_1 \\ \underline{v}_2 \end{bmatrix} (\textit{impedance form}), \qquad \text{Eq. (2-6).}$$

Table 2-2 shows the four-pole parameters in chain form for certain ideal components or assemblies.

Table 2-2. *Four-pole parameters of idealized components.*

Component	Impedance	Symbols
Mass	$\left[\underline{\underline{A}}\right] = \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix}$	m : mass
Spring	$\left[\underline{\underline{A}}\right] = \begin{bmatrix} 1 & 0 \\ \frac{j\omega}{k} & 1 \end{bmatrix}$	k : spring constant
Viscous damper	$\left[\underline{\underline{A}}\right] = \begin{bmatrix} 1 & 0 \\ \frac{1}{c} & 1 \end{bmatrix}$	c : damping coefficient
Oscillator (design, see below)	$\left[\underline{\underline{A}}\right] = \begin{bmatrix} 1 & j\omega m \\ \frac{1}{c + \frac{k}{j\omega}} & \frac{j\omega m + c + \frac{k}{j\omega}}{c + \frac{k}{j\omega}} \end{bmatrix}$	

Table 2-2 *(continued).*

Component	Impedance	Symbols
Absorber (design, see below)	$[\underline{A}] = \begin{bmatrix} 1 & \dfrac{\dfrac{j\omega m}{j\omega m + c + \dfrac{k}{j\omega}}}{c + \dfrac{k}{j\omega}} \\ 0 & 1 \end{bmatrix}$	

The mechanical configuration of the oscillator and the absorber is shown by the following two diagrams (Figures 2-6 and 2-7).

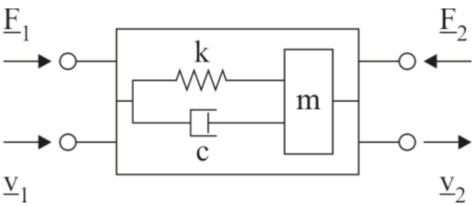


Figure 2-6. *Four-pole representation of an oscillator.*

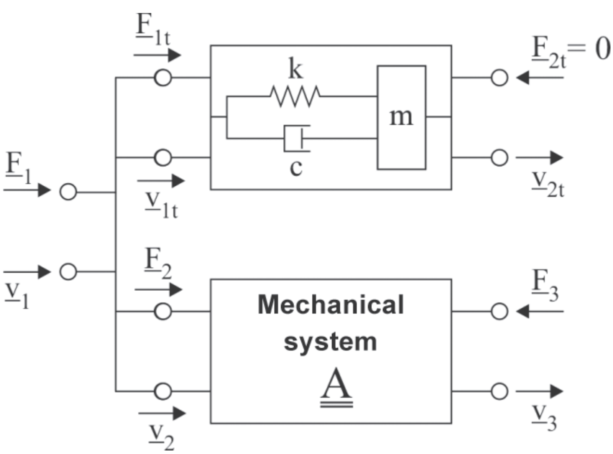


Figure 2-7. *Four-pole representation of an absorber.*

2.3.3 Coupling of four-pole systems

One of the major advantages of four-pole modeling is that is allows easy mathematical treatment of an entire network of mechanical components. In order to take coupling within the network into account, each subsystem may be described in terms of a four-pole system. These are combined mathematically to form an overall four-pole system, allowing the modeling of complex structures. It is then possible to calculate the effectiveness of absorbers or other structures more easily than using differential equations.

In the case of parallel connections, all the interconnected components must be exposed to the same vibration frequency. If subsystems are connected in series, the appropriate four-pole systems must be presented in chain form as the output variables of each four-pole system in the series are also the input variables for the next four-pole system. The subsystems are exposed to the same flow of force and the overall chain matrix $\underline{\underline{A}}_{\text{total}}$ is given by

$$\underline{\underline{A}}_{\text{total}} = \prod_{i=1}^{n} \underline{\underline{A}}^{i}, \qquad \text{Eq. (2-7).}$$

If the input values of several subsystems are rigidly linked, they have identical velocities and the sum of the forces on the input and output side represents the total force in each case (parallel configuration); therefore the overall impedance matrix $\underline{\underline{Z}}_{\text{total}}$ is given by

$$\underline{\underline{Z}}_{\text{total}} = \sum_{i=1}^{n} \underline{\underline{Z}}^{i}, \qquad \text{Eq. (2-8).}$$

Molloy [2-2] has developed a set of equations allowing *n* four-pole systems to be connected in parallel in chain form. This means that it is not necessary to switch between chain form and impedance form:

$$\underline{\underline{A}}_{\text{total}} = \begin{bmatrix} \underline{a}_{11}^{\text{total}} = \dfrac{A}{B} & \underline{a}_{12}^{\text{total}} = \dfrac{AC}{B} - B \\ \underline{a}_{21}^{\text{total}} = \dfrac{1}{B} & \underline{a}_{22}^{\text{total}} = \dfrac{C}{B} \end{bmatrix} \qquad \text{Eq. (2-9),}$$

where $A = \sum_{i=1}^{n} \left(\dfrac{\underline{a}_{11}^{i}}{\underline{a}_{21}^{i}} \right)$, $B = \sum_{i=1}^{n} \left(\dfrac{1}{\underline{a}_{21}^{i}} \right)$, $C = \sum_{i=1}^{n} \left(\dfrac{\underline{a}_{22}^{i}}{\underline{a}_{21}^{i}} \right)$.

Figure 2-8 shows two four-pole systems $\underline{\underline{A}}^1$ and $\underline{\underline{A}}^2$ connected via force $\underline{F}_2$ and velocity $\underline{v}_2$.

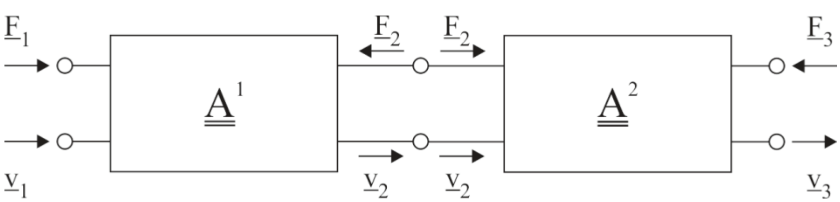


Figure 2-8. *Two mechanical four-pole systems connected in series.*

The two four-pole systems are available in chain form and can therefore be combined to form an overall chain matrix $\underline{\underline{A}}_{\text{total}}$ using equation (2-7):

$$\begin{bmatrix} \underline{F}_1 \\ \underline{v}_1 \end{bmatrix} = \begin{bmatrix} \underline{a}_{11}^{\text{total}} & \underline{a}_{12}^{\text{total}} \\ \underline{a}_{21}^{\text{total}} & \underline{a}_{22}^{\text{total}} \end{bmatrix} \cdot \begin{bmatrix} \underline{F}_3 \\ \underline{v}_3 \end{bmatrix}, \qquad \text{Eq. (2-10).}$$

In explicit form, the following equation applies to the overall chain matrix:

$$\underline{\underline{A}}_{\text{total}} = \begin{bmatrix} \underline{a}_{11}^{\text{total}} & \underline{a}_{12}^{\text{total}} \\ \underline{a}_{21}^{\text{total}} & \underline{a}_{22}^{\text{total}} \end{bmatrix} = \begin{bmatrix} \underline{a}_{11}^{1}\underline{a}_{11}^{2} + \underline{a}_{12}^{1}\underline{a}_{21}^{2} & \underline{a}_{11}^{1}\underline{a}_{12}^{2} + \underline{a}_{12}^{1}\underline{a}_{22}^{2} \\ \underline{a}_{21}^{1}\underline{a}_{11}^{2} + \underline{a}_{22}^{1}\underline{a}_{21}^{2} & \underline{a}_{21}^{1}\underline{a}_{12}^{2} + \underline{a}_{22}^{1}\underline{a}_{22}^{2} \end{bmatrix},$$ Eq. (2-11).

2.3.4 Isolation calculations using four-pole systems

The modeling method presented above using mechanical four-pole systems is especially well-suited for calculating isolating effects.

In general, it is beneficial to take vibration control action as near as possible to the source of the vibration. The action may reduce the sound energy emitted by the source or reflect the sound energy back to the source with a view to increasing damping by causing the sound to pass repeatedly through sound-absorbing structural elements. Structural elements suitable for reducing vibration may include heavy masses and soft springs with and without damping, as well as combinations of such components. In order to quantify the isolating effect of individual elements, two main "loss" parameters are used. The *transmission loss* is the ratio of power or speed upstream from and downstream from an isolating element. The *insertion loss* is a far more effective parameter for describing the effect of isolating elements. It is the ratio of the power and velocity at the "receiving end" of a structure with the isolating element in place to the same value without the isolating element. This value gives a direct indication of the effects of the change in isolation on the dynamics of the overall system.

2.3.4.1 Transmission loss

The following equations for loss values may be derived from the basic equation for mechanical four-pole systems:

$$\underline{F}_1 = \underline{a}_{11}\underline{F}_2 + \underline{a}_{12}\underline{v}_2$$ Eq. (2-12);

$$\underline{v}_1 = \underline{a}_{21}\underline{F}_2 + \underline{a}_{22}\underline{v}_2$$ Eq. (2-13).

In the case of transmission loss, it is necessary to distinguish two values.

2.3.4.2 Transmission loss with reference to velocity

With the terminating impedance

$$\underline{Z}_{\text{t}} := \frac{\underline{F}_2}{\underline{v}_2}$$ Eq. (2-14),

the transmission loss with reference to velocity may be derived from equation (2-13)

$$\underline{D}_{\text{dv}} := \frac{\underline{v}_1}{\underline{v}_2} = \underline{a}_{22} + \underline{a}_{21}\underline{Z}_{\text{t}},$$ Eq. (2-15).

Loss is often expressed as a level (in decibels):

$$\Delta L_{\mathrm{dv}} := 20 \log\left|\frac{\underline{v}_1}{\underline{v}_2}\right| dB, \qquad \text{Eq. (2-16).}$$

If transmission loss is used to describe isolation properties, the fact that the insertion of an oscillating element may lead to a significant increase in velocity $\underline{v}_1$ compared with the situation with non-elastic mounting is ignored. As a result, the values which are calculated or measured are often too favorable, depending on the measurement configuration. Nevertheless, as the transmission loss is easy to measure and calculate, it is often used for assessment.

2.3.4.3 Transmission loss with reference to force

With the terminating impedance, the transmission loss with reference to force may be derived from equation (2-12) as follows:

$$\underline{D}_{\mathrm{dF}} := \frac{\underline{F}_1}{\underline{F}_2} = \underline{a}_{11} + \frac{\underline{a}_{12}}{\underline{Z}_{\mathrm{t}}}, \qquad \text{Eq. (2-17).}$$

The loss expressed as a level is given by

$$\Delta L_{\mathrm{dF}} := 20 \log\left|\frac{\underline{F}_1}{\underline{F}_2}\right| \mathrm{dB}, \qquad \text{Eq. (2-18).}$$

In contrast to the transmission loss with reference to velocity, it is difficult to make direct measurements of the transmission loss with reference to force, because instruments need to be inserted into the flow of force. For the calculation of transmission loss, only two four-pole parameters and the terminating impedance are needed, However, for the calculation of values with reference to velocity and force, all the four-pole parameters and the terminating impedance are required. In this case, the source impedance is irrelevant.

2.3.4.4 Insertion loss

Where vibration control measures are taken, level values upstream and downstream from the isolator are of secondary importance. It is more important to know what vibration arrives at the receiving end with an isolating component, compared to the situation without the isolating component. This is the only way of assessing whether the use of the isolating component is actually beneficial. If the velocity at the output of the original system is designated as $\underline{v}_2$ and the velocity at the output of the system with isolation is designated as $\underline{v}_2'$, the insertion loss is given by

$$\underline{D}_{\mathrm{e}} := \frac{\underline{F}_2}{\underline{F}_2'} = \frac{\underline{v}_2}{\underline{v}_2'} = \frac{\underline{a}_{22}'\underline{Z}_{\mathrm{i}} + \underline{a}_{21}'\underline{Z}_{\mathrm{i}}\underline{Z}_{\mathrm{t}} + \underline{a}_{12}' + \underline{a}_{11}'\underline{Z}_{\mathrm{t}}}{\underline{a}_{22}\underline{Z}_{\mathrm{i}} + \underline{a}_{21}\underline{Z}_{\mathrm{i}}\underline{Z}_{\mathrm{t}} + \underline{a}_{12} + \underline{a}_{11}\underline{Z}_{\mathrm{t}}}, \qquad \text{Eq. (2-19).}$$

In equation (2-19), the primed variables represent the velocity with isolation or the four-pole parameters of the isolating element. The unprimed variables represent the velocity without isolation and the four-pole parameters of the original configuration. In the event that the original situation corresponds to a "short circuit" between the source and terminating impedance, e.g. with non-elastic machine mounting, equation (2-19) is reduced to

$$\underline{D}_e := \frac{\underline{F}_2}{\underline{F}_2'} = \frac{\underline{v}_2}{\underline{v}_2'} = \frac{\underline{a}_{22}'\underline{Z}_i + \underline{a}_{21}'\underline{Z}_i\underline{Z}_t + \underline{a}_{12}' + \underline{a}_{11}'\underline{Z}_t}{\underline{Z}_i + \underline{Z}_t}, \qquad \text{Eq. (2-20).}$$

$\underline{Z}_i$ designates the input impedance of the source structure and $\underline{Z}_t$ the input impedance of the receiving structure. $\underline{Z}_i$ is also referred to as source impedance and $\underline{Z}_t$ as terminating impedance. Another advantage of using insertion loss is that there is no difference between the loss with respect to force and to velocity, because there is a fixed ratio between the output force and velocity as a result of the terminating impedance. In order to calculate the insertion loss, it is necessary to have full information, including all four-pole parameters and the source and terminating impedances. In contrast, it is very easy to determine the insertion loss by measuring the velocities at the receiving end with the original configuration and the isolating component inserted and then dividing the two values. *Seidel* [2-3] also uses the reciprocals of the loss values. The reciprocal of the insertion loss is the insertion transmission ratio:

$$\underline{T}_e := \frac{1}{\underline{D}_e} = \frac{\underline{F}_2'}{\underline{F}_2} = \frac{\underline{v}_2'}{\underline{v}_2}, \qquad \text{Eq. (2-21).}$$

The advantage of this approach is that critical points with low isolation appear as peaks in the curve and can therefore easily be identified.

2.3.4.5 Example: shock absorber top mount for a car suspension

In addition to springs, car chassis are equipped with shock absorbers intended to reduce movement at the eigenfrequencies of the body and the wheel. This is necessary in order to improve safety. To optimize the acoustics of the vehicle interior, elastic mounts or "top mounts" are installed above the shock absorber. The following paragraphs give an insertion loss calculation based on a simple model to indicate the change in the vibration transmitted to the vehicle body as a result of the use of this component (Figure 2-9). It should be pointed out that this is a highly simplified model. For example, non-linear damping behavior, especially friction in the shock absorber, which is acoustically relevant, is not taken into consideration.

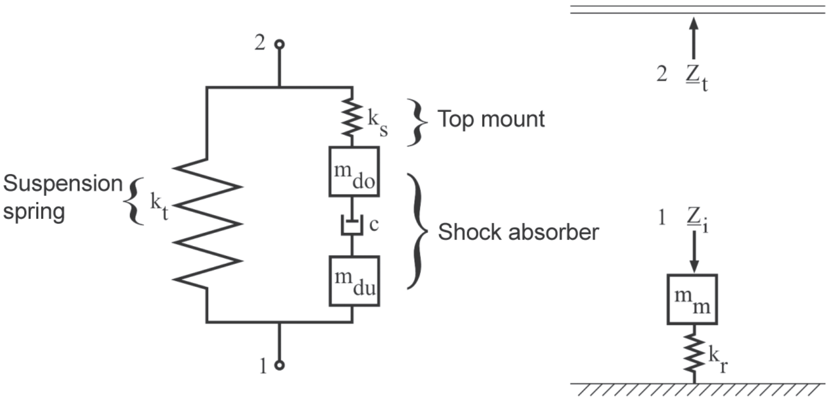


Figure 2-9. *Modeling of a car suspension.*

In Figure 2-9, the model for a suspension strut is shown on the left-hand side. In parallel to the suspension spring k_t, which bears the weight of the vehicle, the shock absorber and top mount k_s are installed in series. The shock absorber is modeled by a bottom mass, m_{du}, an ideal damper, c, and a top mass, m_{do}, (piston rod). The overall system of the suspension strut is calculated with and without the top mount. The source impedance $\underline{Z}_i$ is calculated from the wheel stiffness k_r and the mass of the wheel carrier and wheel moving with the strut m_m. For the terminating impedance $\underline{Z}_t$ plate behavior instead of the behavior of the vehicle body is assumed.

The results of the calculation are shown in Figure 2-10. In the bottom graph it can be seen that the acoustic benefits from about 30 Hz upwards are made possible by significantly reduced isolation at and around the eigenfrequency of the wheel (14 Hz). In this range, vibration levels are higher with the top mount than if the shock absorber were rigidly connected to the body. The insertion loss values are lower than 0 dB. As an undesirable effect, less energy is dissipated in the shock absorber as a result of the softer shock absorber connection. If wheel hop is to be avoided, it is important to ensure that the top mount is not too soft. This severely simplified model already indicates the conflicting objectives faced in the design of top mounts. In practice, a non-linear stiffness plot is produced by using stops. As a result, the mount is very soft at acoustically relevant small displacement values. In the event of severe displacement, for example as a result of wheel hop, the top mount becomes harder, ensuring that as much energy as possible is dissipated in the shock absorber. The top half of Figure 2-10 shows a phase plot of insertion loss. The phase position at the body input is changed by the top mount as the shock absorber assembly with the mount is dominated by this component. Without a top mount, the phase angle of the shock absorber, which has a maximum shift of 90°, is established.

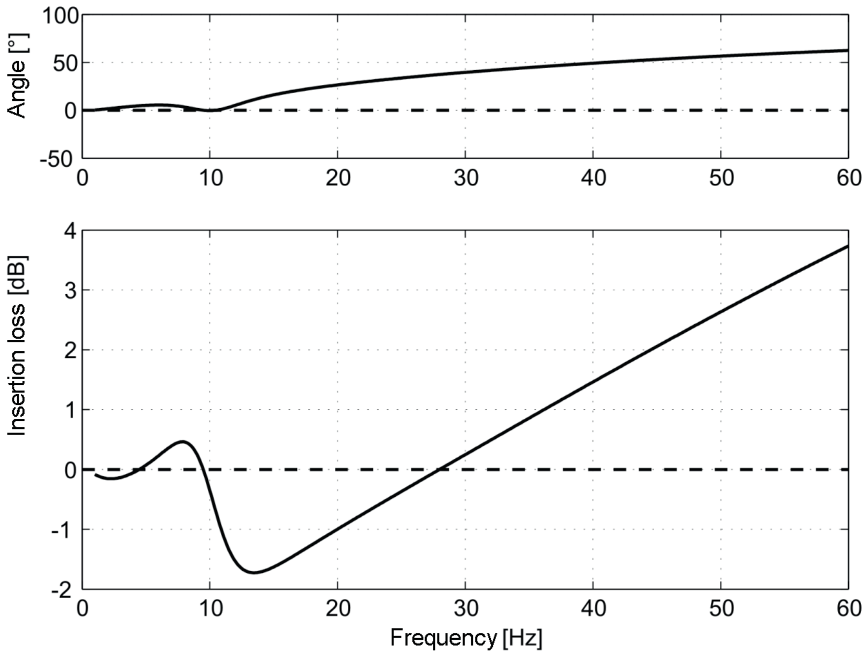


Figure 2-10. *Insertion loss of a top mount.*

2.4 Effects of damping and friction on isolation

2.4.1 Introduction

The preceding section demonstrates the approach of describing the vibration isolation of a rubber mount using an idealized spring-mass system. This approach assumes ideal spring properties and a rigid environment. The mathematical description based on this approach results in the transmission of vibration excitation shown in the graph in Figure 2-11. Depending on the tuning of the spring-mass system and the frequency of the oscillating exciting force, the result is the amplification or isolation of harmonic excitation. The example shows an eigenfrequency of 10 Hz set via the spring rate and the mass. In the frequency window between 10 and 20 Hz, vibration transmission is sensitive to fluctuations in the spring characteristics. Even small changes can determine whether the vibration is isolated or amplified and may even bring the system close to resonant excitation. Only higher excitation frequencies with a greater margin from the eigenfrequency, from 20 Hz upwards in this example, result in stable isolation with a curve that falls off steadily as a straight line.

In this frequency range, each doubling of the frequency (octave) results in a constant gain of 12 dB in isolation on the basis of an idealized mathematical analysis.

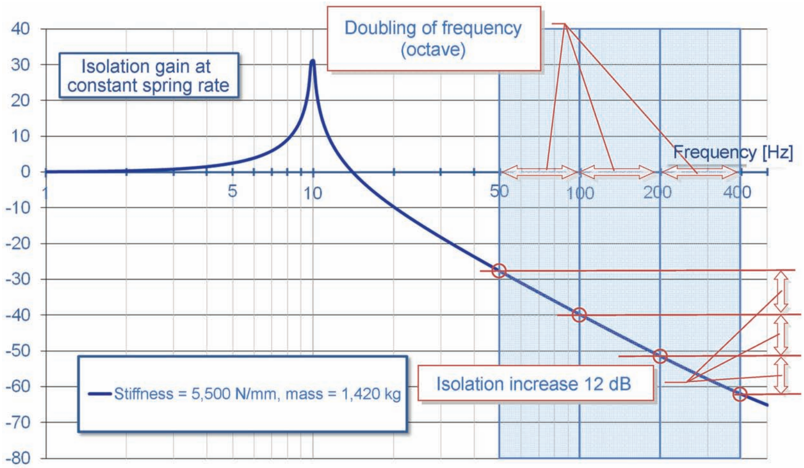


Figure 2-11. *Transmission behavior of a spring mass system (isolation increase 12 dB per octave, i.e. doubling of frequency).*

On the same theoretical basis, this simplified mathematical approach also describes the effect of different spring rates on the isolation effect. A stiffer spring leads to a shift in the eigenfrequency towards higher values while a reduction in spring stiffness has the opposite effect. The left-hand section of the transmission graph is stretched or compressed in the X direction, while the continuous linear increase in isolation on the right-hand side is shifted in parallel (Figure 2-12).

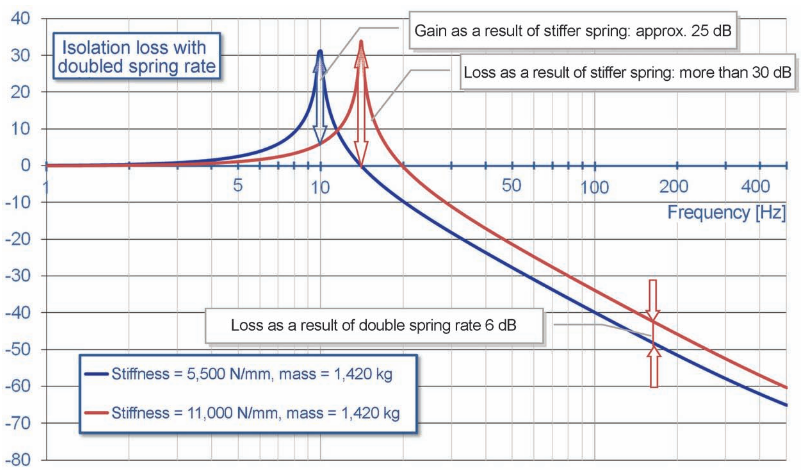


Figure 2-12. *A stiffer spring increases the eigenfrequency. In the critical frequency window, this results in a dramatic isolation loss; in the supercritical range, isolation is reduced to 6 dB.*

The depiction gives some orientation: Each doubling of the spring rate (from 5,500 N/mm to 11,000 N/mm in the example) results in an isolation loss of 6 dB. At the same time, the eigenfrequency rises by a factor of 1.41 (= $\sqrt{2}$). In this frequency range, the isolation losses may be dramatically higher.

Up to this point, the analysis is based on the behavior of an idealized spring.